Comparison of Upward and Downward Generalization in CF-induction

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Abstract. CF-induction is a sound and complete procedure for finding hypotheses in full clausal theories. It is based on the principle of Inverse Entailment (IE), and consists of two procedures: construction of a bridge theory and generalization of it. There are two possible ways to realize the generalization task in CF-induction. The one uses a single deductive operator, called γ-operator, and the other uses a recently proposed form of inverse subsumption. Whereas both are known to retain the completeness of CF-induction, their favorite hypotheses, which tend to be found, are different from each other. In this paper, we introduce those two generalization approaches in CF-induction, and then investigate their logical relationship and features especially on each favorite hypotheses.
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1 Introduction

CF-induction [1] is one of the modern explanatory ILP methods based on the principle of *Inverse Entailment* (IE) [4]. Given a background theory B and examples E, the task of explanatory induction is to find a hypothesis H such that $B \wedge H \models E$ and $B \wedge H$ is consistent. This task is logically equivalent to find a consistent hypothesis H such that $B \wedge \neg E \models \neg H$. Modern IE-based methods then compute hypotheses in two steps: first constructing a bridge theory F_i and next generalizing its negation into a hypothesis H, described as follows:

$$\overline{B \wedge \neg E \models F_1 \models \cdots \models} F_i \models F_{i+1} \models \cdots \models F_n \models \neg H$$

$$\text{(Generalization)}$$

$$\neg F_i \mathrel{=\!\!|} \neg F_{i+1} \mathrel{=\!\!|} \cdots \mathrel{=\!\!|} \neg F_n \mathrel{=\!\!|} H$$

Fig. 1. Hypothesis finding based on inverse entailment

We denote by $\mathrel{=\!\!|}$ the inverse relation of entailment, called *anti-entailment*.

CF-induction first computes some interesting consequences, called *characteristic clauses*, of $B \wedge \neg E$, which satisfy a given language bias and then constructs a bridge theory, often denoted by CC. The bridge theory consists of ground instances from the characteristic clauses. After translating $\neg CC$ into CNF, CF-induction generalizes it to a hypothesis based on anti-entailment.

CF-induction ensures soundness and completeness of finding hypotheses in full clausal theories. Compared with other IE-based methods [4, 5, 3, 6, 3, 7], it has three important benefits [8]. Unlike Progol [4], HAIL [5] and Imparo [3], it enables the solution of more complex problems in richer knowledge representation formalisms beyond Horn logic. Unlike FC-HAIL [6], it is complete for finding full clausal hypotheses. Unlike the residue procedure [7], it can exploit language bias to specify the search space to focus the procedure on some relevant part.

However, the generalization of CF-induction has to handle many highly non-deterministic operators, like *inverse resolution*, to ensure the completeness in the sense that it can find any hypothesis H such that $\neg CC \models\!\!\!\!\mid H$. There are two recent works [8, 9] that can be used to reduce the non-determinisms in generalization. The literature [8] focuses on the entailment relation $CC \models \neg H$ for a bridge theory CC and a ground hypothesis H. It shows that this relation is logically simplified with a single deductive operator $\vdash_\gamma$, called γ-operator, which warrants the insertion of literals into the clauses of CC. The literature [9] shows a technique to reduce the anti-entailment relation $\neg CC \models\!\!\!\!\mid H$ to the anti-subsumption relation[3] $CC^* \preceq H$ where CC^* is a certain clausal theory logically equivalent to $\neg CC$. Hence, there are two approaches for generalization in CF-induction. Hereafter, we call the former (*resp.* the latter) *downward* (*resp. upward*) generalization (See Fig. 2).

(Downward generalization)

$$B \wedge \neg E \models \cdots \models CC \vdash_\gamma CC_1 \vdash_\gamma \cdots \vdash_\gamma CC_n \vdash_\gamma \neg H$$

(Upward generalization)

$$CC^* \preceq CC_1^* \preceq \cdots \preceq CC_n^* \preceq H$$

Fig. 2. Upward and downward generalization in CF-induction

Whereas both are known to retain the completeness of CF-induction, their favorite hypotheses, which tend to be found, are different from each other. In this paper, we introduce these two generalization approaches and investigate their logical relationship and features especially on each favorite hypotheses. Due to space limitations, full proofs are omitted.

2 Notion and terminology

Here, we review the notion and terminology in ILP [10]. A *clause* is a finite disjunction of literals which is often identified with the set of its disjuncts. A clause $\{A_1, \ldots, A_n, \neg B_1, \ldots, \neg B_m\}$, where each A_i, B_j is an atom, is also written as $B_1 \wedge \cdots \wedge B_m \supset A_1 \vee \cdots \vee A_n$. A *Horn clause* is a clause which contains at most one positive literal; otherwise it is a *non-Horn clause*. It is known that a clause is a *tautology* if it has two complementary literals. A *clausal theory* is a

[3] We call the inverse of subsumption *anti-subsumption*, and denote it by $\preceq$.

finite set of clauses. A clausal theory is *full* if it contains at least one non-Horn clause. A clausal theory S is often identified with the conjunction of its clauses and is said to be in *Conjunctive Normal Form* (CNF).

Let C and D be two clauses. C *subsumes* D, denoted $C \succeq D$, if there is a substitution θ such that $C\theta \subseteq D$. C *properly* subsumes D if $C \succeq D$ but $D \not\succeq C$. For a clausal theory S, μS denotes the set of clauses in S not properly subsumed by any clause in S. Let S and T be clausal theories. S (*theory-*) *subsumes* T, denoted by $S \succeq T$, if for every $C \in T$, there is a clause $D \in S$ such that $C \succeq T$.

When S is a clausal theory, the complement of S, denoted by $\overline{S}$, is defined as a clausal theory obtained by translating $\neg S$ into CNF using a standard translation procedure [10]. (In brief, $\overline{S}$ is obtained by converting $\neg S$ into prenex conjunctive normal form with standard equivalence-preserving operations and Skolemizing it.) Note that the complement $\overline{S}$ may contain redundant clauses like tautologies or subsumed ones. Especially, we call the clausal theory consisting of the non-tautological clauses in $\mu\overline{S}$ the *minimal complement* of S, denoted by $M(S)$.

We denote by $\models$ the classical logical entailment relation. Let S and T be clausal theories. S and T are (*logically*) *equivalent*, denoted by $S \equiv T$, if $S \models T$ and $T \models S$. For a clausal theory S, a *consequence* of S is a clause entailed by S. We denote by $Th(S)$ the set of all consequences of S. Note that $\mu Th(S)$ denotes the set of all subsumption-minimal consequences of S.

We give the definition of *hypotheses* in the logical setting of ILP as follows:

Definition 1 (Hypotheses). Let B and E be clausal theories, representing a background theory and (positive) examples, respectively. Then H is a *hypothesis wrt B and E* if and only if H is a clausal theory such that $B \wedge H \models E$ and $B \wedge H$ is consistent. We refer to a "hypothesis" instead of a "hypothesis wrt B and E" if no confusion arises.

3 Upward generalization in CF-induction

Here, we introduce upward generalization of CF-induction. A bridge theory of CF-induction consists of instances from so-called (*new*) *characteristic clauses* [2], which are the subsumption-minimal consequences of $B \wedge \overline{E}$ (that is, $\mu Th(B \wedge \overline{E})$) satisfying a given language bias. In the following, we assume the language bias as follows.

Definition 2 (Induction field). An *induction field*, denoted by $\mathcal{I_H} = \langle \mathbf{L} \rangle$, where $\mathbf{L}$ is a finite set of literals to appear in ground hypotheses. A clausal theory S *belongs to* $\mathcal{I_H}$ if every literal in S is included in $\mathbf{L}$. Note that $\overline{\mathbf{L}}$ is the set of negations of literals in $\mathbf{L}$.

Using the above notion, we can describe bridge theories of CF-induction[4].

[4] Technically, they must contain at least one new characteristic clause of $\overline{E}$ to ensure the consistency with the background theory. See [1] for more detail.

Definition 3 (Bridge theory). Let B and E be a background theory and examples, respectively. Let $\mathcal{I}_{\mathcal{H}} = \langle \mathbf{L} \rangle$ be an induction field. Then a ground clausal theory CC is a *bridge theory of CF-induction* wrt B, E and $\mathcal{I}_{\mathcal{H}}$ if CC consists of instances from characteristic clauses of $B \wedge \overline{E}$, each of which belongs to $\langle \overline{\mathbf{L}} \rangle$. If no confusion arises, a bridge theory of CF-induction wrt B, E and $\mathcal{I}_{\mathcal{H}}$ is simply called a bridge theory.

Theorem 1. Let CC be a bridge theory. Then, for any hypothesis H such that $H \models \neg CC$, there is another bridge theory CC' such that $H \succeq M(CC')$.

Theorem 1 can be obtained by applying the technique in the literature [9] to reduce anti-entailment to anti-subsumption. Based on Theorem 1, any hypothesis can be found over the subsumption lattice bounded by the minimal complement of a bridge theory.

Example 1. [9] Let examples E_1 be $\{defeat(claudius)\}$ and a background theory B_1 be $\{\emptyset\}$. Let a target hypothesis H_1 and an induction field $\mathcal{I}_{\mathcal{H}_1}$ be as follows:

$$H_1 = \{risk\_life(hamlet),\ risk\_life(hamlet) \supset defeat(claudius)\}.$$
$$\mathcal{I}_{\mathcal{H}_1} = \{defeat(claudius), risk\_life(hamlet), \neg risk\_life(hamlet)\}.$$

Suppose that a bridge theory $CC_1 = \{\neg defeat(claudius)\}$ is given. We notice that $H_1 \models \neg CC_1$ but $H_1 \not\succeq \neg CC_1$. Next, consider another bridge theory $CC_2 = CC_1 \cup \{risk\_life(hamlet) \vee \neg risk\_life(hamlet)\}$. Note that the bridge theories of CF-induction are allowed to include tautologies belonging to the induction field. $M(CC_2)$ is $\{\ risk\_life(hamlet) \vee\ defeat(claudius),\ risk\_life(hamlet) \supset defeat(claudius)\}$. Indeed, H subsumes $M(CC_2)$.

4 Downward generalization in CF-induction

Now, we focus on the entailment relation $CC \models \neg H$ for a bridge theory CC and a ground hypothesis H. Recall that the bridge theory consists of subsumption-minimal consequences of $B \wedge \overline{E}$. Hence, constructing a relevant bridge theory may be viewed as approximating $\neg H$ with some relevant (subsumption-minimal) consequences of $B \wedge \overline{E}$. In the literature [8], the entailment relation has been logically simplified with the following deductive operator:

Definition 4 (γ-operator). Let S and T be clausal theories. T is *directly γ-derivable* from S if and only if T is obtained from S under the following condition:

$$T = (S - \{D\}) \cup \{C_1, \ldots, C_n\} \text{ for some } n \geq 0 \text{ where } C_i \supseteq D \text{ for all } 1 \leq i \leq n.$$

We write $S \vdash_{\gamma} T$ if and only if T is directly γ-derivable from S and $\vdash_{\gamma}^{*}$ is a reflexive and transitive closure of $\vdash_{\gamma}$.

Theorem 2. [8] Let CC be a bridge theory. For any ground hypothesis H such that $CC \models \neg H$, there is another bridge theory CC' such that $CC' \vdash_{\gamma}^{*} M(H)$.

By Theorem 2, any ground hypothesis can be derived by γ-operator. Recall Example 1. We can obtain $M(H_1)$ from CC_2 in such a way that the unit clause $\neg defeat(claudius)$ in CC_2 is expanded to $\neg defeat(claudius) \vee \neg risk\_life(hamlet)$ and replaced by it. We remark that any non-ground hypothesis can be also obtained by applying the γ-operator followed by anti-instantiation [8].

Example 2. Let a background theory B_2 and examples E_2 be as follows:

$$B_2 = \{female(s) \vee male(s)\}.\ E_2 = \{human(s)\}.$$

Consider a bridge theory $CC_3 = \{female(s) \vee male(s),\ \neg human(s)\}$. Suppose that we construct $M(H_2)$ by applying γ-operator to CC_3 in such a way that $\neg human(s)$ is expanded to two clauses $\neg human(s) \vee female(s)$ and $\neg human(s) \vee male(s)$ and replaced by them. Then, we obtain H_2 by computing $M(M(H_2))$:

$$\{\neg female(s) \vee human(s),\ \neg male(s) \vee human(s),\ \neg female(s) \vee \neg male(s)\}.$$

By applying an anti-instantiation operator, we obtain a non-ground hypothesis: $\{female(X) \supset human(X),\ male(X) \supset human(X),\ \neg female(X) \vee \neg male(X)\}$. Note that the last clause $\neg female(X) \vee \neg male(X)$ does not involve in explaining E. In this sense, this clause can be treated as a redundant one, though it is a correct integrity constraint on $female(X)$ and $male(X)$.

5 Discussion and concluding remarks

In this paper, we studied two generalization approaches in CF-induction. Both approaches are based on the subsumption relation and retain the completeness of CF-induction. However, their favorite hypotheses, which tend to be found, are different from each other. The difference lies in their target theories to be searched: on the one hand, downward generalization focuses on the minimal complement of a hypothesis $M(H)$. On the other hand, upward generalization focuses on a hypothesis H itself. Fig. 3 describes their search strategies.

The upward approach is based on anti-subsumption, and directly computes hypotheses from $M(CC)$. Hence, it is more likely to find *compressed* hypotheses, since it uses *anti-subsumption* operators dropping some literals from a clause. In contrast, the downward approach is likely to find *officious* hypotheses, which are not necessarily used to explain the examples (but consistent with the background theory), like Example 2. To derive such hypotheses in upward generalization, we may need *anti-weakening* operators adding some clauses.

In the inductive learning point of view, we are used to seek more compressed descriptions based on the principle of Occum's razor. Thus, the upward approach is suitable for this principle. In contrast, the downward approach interestingly takes the risk that hypotheses can contain some *extra* rules that are not necessary for explaining examples. In some cases, this efficiently works for giving users some unexpected insights to the incomplete background theory.

Our result enables CF-induction to bi-directionally search for the target hypothesis in both downward and upward approaches. The upward approach has

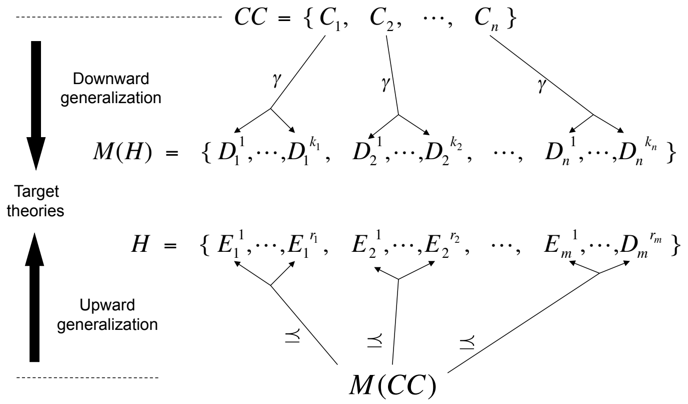

Fig. 3. Search strategies in upward and downward approaches

been actively studied in the context of *refinement operators*, and thus we can use their sophisticated operators, such as heuristics-based ones. However, the downward approach is not straightforward, since the compression measure is not simply used. Note that the minimal complement $M(H)$ is related to the models in H. Hence, it would be helpful if there is some relevant measure for evaluating the models.

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