Can ILP Deal with Incomplete and Vague Structured Knowledge?

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1 Introduction

Ontologies are currently a preminent source of *structured* knowledge. The logical languages known as *Description Logics* (DLs) [1] play a key role in the design of ontologies as they are essentially the theoretical counterpart of the *Web Ontology Language OWL 2* [3] - the current standard language to represent ontologies - and its profiles. [4] E.g., DL-Lite [2] is the DL behind the *OWL 2 QL* profile and is especially aimed at applications that use very large volumes of instance data, and where query answering is the most important reasoning task.

Incompleteness and *vagueness* are inherent properties of knowledge in several real world domains. DL-based ontology languages were born to address the former. Indeed, the Open World Assumption (OWA) holds in DLs. Fuzzy extensions of DLs have been more recently devised to address the latter (see the survey in [7]). They include, among others, a fuzzy DL-Lite like DL [12] which has been implemented in the SoftFacts system [5].

In this abstract, we sketch the results of our preliminary investigation of the issue of whether ILP can deal with incomplete and vague structured knowledge. More precisely, we provide the ingredients for learning fuzzy DL inclusion axioms with ILP. The resulting method adapts known results in ILP concerning the induction of crisp rules, notably FOIL [8], to the novel context of ontologies.

The abstract is organized as follows. Section 2 introduces fuzzy DLs. Section 3 describes our preliminary contribution to the problem in hand, also by means of an illustrative example. Section 4 concludes the paper with final remarks and comparison with related work.

2 Fuzzy Description Logics

For computational reasons, the logic we adopt is based on a fuzzy extension of the DL-Lite DL without negation [12]. It supports at the intentional level unary relations (called *concepts*) and binary relations (called *roles*), while supports n-ary relations (relational tables) at the extensional level.

[3] http://www.w3.org/TR/2009/REC-owl2-overview-20091027/
[4] http://www.w3.org/TR/owl2-profiles/.
[5] See, http://www.straccia.info/software/SoftFacts/SoftFacts.html

Formally, a *knowledge base* $\mathcal{K} = \langle \mathcal{F}, \mathcal{O}, \mathcal{A} \rangle$ consists of a *facts component* $\mathcal{F}$, an *ontology component* $\mathcal{O}$ and an *abstraction component* $\mathcal{A}$. Information can be retrieved from $\mathcal{K}$ by means of an appropriate *query language*.

In order to deal with vagueness, Gödel logic is adopted, where $a \otimes b = \min(a, b)$, $a \oplus b = \max(a, b)$, $\ominus\, a = \begin{cases} 1 & \text{if } a \leqslant b \\ b & \text{otherwise} \end{cases}$, and $a \Rightarrow b = \begin{cases} 1 & \text{if } a = 0 \\ 0 & \text{otherwise} \end{cases}$.

For a detailed account of the semantics, see [11].

Facts Component. $\mathcal{F}$ is a finite set of expressions of the form

$$R(c_1, \ldots, c_n)[s] \ , \tag{1}$$

where R is an n-ary relation, every c_i is a constant, and s is a degree of truth (or *score*) in $[0, 1]$ indicating to which extent the tuple $\langle c_1, \ldots, c_n \rangle$ is an instance of relation R. We may omit the score component and in such case the value 1 is assumed. Facts are stored in a relational database.

Ontology Component. $\mathcal{O}$ is a finite set of *inclusion axioms* having the form

$$Rl_1 \sqcap \ldots \sqcap Rl_m \sqsubseteq Rr \ , \tag{2}$$

where $m \geqslant 1$, all Rl_i and Rr have the same arity and each Rl_i is a so-called *left-hand relation* and Rr is a *right-hand relation*. We assume that relations occurring in $\mathcal{F}$ do not occur in inclusion axioms (so, we do not allow that database relation names occur in $\mathcal{O}$). The intuitive semantics is that if a tuple $\mathbf{c}$ is instance of each relation Rl_i to degree s_i then $\mathbf{c}$ is instance of Rr to degree $\min(s_1, \ldots, s_m)$.

The exact syntax of the relations appearing on the left-hand and right-hand side of inclusion axioms is specified below:

$$\begin{array}{l} Rl \longrightarrow A \mid R[i_1, i_2] \\ Rr \longrightarrow A \mid R[i_1, i_2] \mid \exists R.A \end{array} \tag{3}$$

where A is an atomic concept and R is a role with $1 \leqslant i_1, i_2 \leqslant 2$. Here $R[i_1, i_2]$ is the projection of the relation R on the columns i_1, i_2 (the order of the indexes matters). Hence, $R[i_1, i_2]$ has arity 2. Additionally, $\exists R.A$ is a so-called qualified existential quantification on roles which corresponds to the FOL formula $\exists y.R(x, y) \land A(y)$ where $\land$ is interpreted as the t-norm $\otimes$ in the Gödel logic.

Abstraction Component. $\mathcal{A}$ is a finite set of *abstraction statements* of the form

$$R \mapsto (c_1, \ldots, c_n)[c_{score}].sql \ , \tag{4}$$

where *sql* is a SQL statement returning n-ary tuples $\langle c_1, \ldots, c_n \rangle$ $(n \leqslant 2)$ with score determined by the c_{score} column. The tuples have to be ranked in decreasing order of score and, as for the fact component, we assume that there cannot be two records $\langle \mathbf{c}, s_1 \rangle$ and $\langle \mathbf{c}, s_2 \rangle$ in the result set of *sql* with $s_1 \neq s_2$ (if there are, then we remove the one with the lower score). The score c_{score} may be omitted and in that case the score 1 is assumed for the tuples. We assume that R occurs in $\mathcal{O}$, while all of the relational tables occurring in the SQL statement occur in $\mathcal{F}$. Finally, we assume that there is at most one abstraction statement for each abstract relational symbol R.

Query Language. The query language enables the formulation of conjunctive queries with a scoring function to rank the answers. More precisely, a *ranking query* is of the form

$$\begin{aligned} q(\mathbf{x})[s] \leftarrow \exists \mathbf{y}\ & R_1(\mathbf{z}_1)[s_1], \ldots, R_l(\mathbf{z}_l)[s_l], \\ & \mathsf{OrderBy}(s = f(s_1, \ldots, s_l, p_1(\mathbf{z}'_1), \ldots, p_h(\mathbf{z}'_h)) \end{aligned} \tag{5}$$

where

1. q is an n-ary relation, every R_i is a n_i-ary relation ($1 \leqslant n_i \leqslant 2$). $R_i(\mathbf{z_i})$ may also be of the form $(z \leqslant v), (z < v), (z \geqslant v), (z > v), (z = v), (z \neq v)$, where z is a variable, v is a value of the appropriate concrete domain;
2. $\mathbf{x}$ are the *distinguished variables.*
3. $\mathbf{y}$ are existentially quantified variables called the *non-distinguished variables.* We omit to write $\exists \mathbf{y}$ when $\mathbf{y}$ is clear from the context;
4. $\mathbf{z_i}, \mathbf{z'_j}$ are tuples of constants or variables in $\mathbf{x}$ or $\mathbf{y}$;
5. $s, s_1, \ldots, s_l$ are distinct variables and different from those in $\mathbf{x}$ and $\mathbf{y}$;
6. p_j is an n_j-ary *fuzzy predicate* assigning a score $p_j(\mathbf{c}_j) \in [0, 1]$ to each n_j-ary tuple $\mathbf{c}_j$ of constants.
7. f is a *scoring function* $f\colon ([0, 1])^{l+h} \to [0, 1]$, which combines the scores of the l relations $R_i(\mathbf{c}'_i)$ and the n fuzzy predicates $p_j(\mathbf{c}''_j)$ into an overall score s to be assigned to $q(\mathbf{c})$.

We call $q(\mathbf{x})[s]$ its *head*, $\exists \mathbf{y}.R_1(\mathbf{z}_1)[s_1], \ldots, R_l(\mathbf{z}_l)[s_l]$ its *body* and $\mathsf{OrderBy}(s = f(s_1, \ldots, s_l, p_1(\mathbf{z}'_1), \ldots, p_h(\mathbf{z}'_h))$ the *scoring atom.* We also allow the scores $[s]$, $[s_1]$, $\ldots$, $[s_l]$ and the scoring atom to be omitted. In this case we assume the value 1 for s_i and s instead. The informal meaning of such a query is: if $\mathbf{z}_i$ is an instance of R_i to degree at least or equal to s_i, then $\mathbf{x}$ is an instance of q to degree at least or equal to s, where s has been determined by the scoring atom.

The answer set $ans_{\mathcal{K}}(q)$ over $\mathcal{K}$ of a query q is the set of tuples $\langle \mathbf{t}, s \rangle$ such that $\mathcal{K} \models q(\mathbf{t})[s]$ with $s > 0$ (informally, $\mathbf{t}$ satisfies the query to non-zero degree s) and the score s is as high as possible, *i.e.* if $\langle \mathbf{t}, s \rangle \in ans_{\mathcal{K}}(q)$ then (i) $\mathcal{K} \not\models q(\mathbf{t})[s']$ for any $s' > s$; and (ii) there cannot be another $\langle \mathbf{t}, s' \rangle \in ans_{\mathcal{K}}(q)$ with $s > s'$.

3 ILP for Learning Fuzzy DL Inclusion Axioms

In this section we consider a learning problem where:

- the target concept H is a DL-Lite atomic concept;
- the background theory $\mathcal{K}$ is a DL-Lite like knowledge base $\langle \mathcal{F}, \mathcal{O}, \mathcal{A} \rangle$ of the form described in Section 2;
- the training set $\mathcal{E}$ is a collection of fuzzy DL-Lite like facts of the form (1) and labeled as either positive or negative examples for H. We assume that $\mathcal{F} \cap \mathcal{E} = \emptyset$;
- the target theory $\mathcal{H}$ is a set of inclusion axioms of the form

$$B \sqsubseteq H \tag{6}$$

where H is an atomic concept, $B = C_1 \sqcap \ldots \sqcap C_m$, and each concept C_i has syntax

$$C \longrightarrow A \mid \exists R.A \mid \exists R.\top\ . \tag{7}$$

We now show how we may learn inclusion axioms of the form (6). To this aim, we define for $C \neq H$

$$\mathcal{I}_{ILP} \models C(t) \text{ iff } \mathcal{K} \cup \mathcal{E} \models C(t)[s] \text{ and } s > 0 \; . \tag{8}$$

That is, we write $\mathcal{I}_{ILP} \models C(t)$ iff it can be inferred from $\mathcal{K}$ and $\mathcal{E}$ that t is an instance of concept C to a non-zero degree.

Now, in order to account for multiple fuzzy instantiations of fuzzy predicates occurring in the inclusion axioms of interest to us, we propose the following formula for computing the *confidence degree* of an inclusion axiom:

$$cf(B \sqsubseteq H) = \frac{\sum_{t \in P} B(t) \Rightarrow H(t)}{|D|} \tag{9}$$

where

- $P = \{t \mid \mathcal{I}_{ILP} \models C_i(t) \text{ and } H(t)[s] \in \mathcal{E}^+\}$, *i.e.* P is the set of instances for which the implication covers a positive example;
- $D = \{t \mid \mathcal{I}_{ILP} \models C_i(t) \text{ and } H(t)[s] \in \mathcal{E}\}$, *i.e.* D is the set of instances for which the implication covers an example (either positive or negative);
- $B(t) \Rightarrow H(t)$ denotes the degree to which the implication holds for the instance t;
- $B(t) = \min(s_1, \ldots, s_n)$, with $\mathcal{K} \cup \mathcal{E} \models C_i(t)[s_i]$;
- $H(t) = s$ with $H(t)[s] \in \mathcal{E}$.

Clearly, the more positive instances supporting the inclusion axiom, the higher the confidence degree of the axiom.

Note that the confidence score can be determined easily by submitting appropriate queries via the query language described in Sect. 2. More precisely, proving the fuzzy entailment in (8) for each C_i is equivalent to answering a unique ranking query whose body is the conjunction of the relations R_l resulting from the transformation of C_i's into FOL predicates and whose score s is given by the minimum between s_l's.

For illustrative purposes we consider the following case involving the classification of hotels as good ones. We assume to have a background theory $\mathcal{K}$ with a relational database $\mathcal{F}$ storing facts such as

HotelTable		
id	rank	noRooms
h1	3	21
h2	5	123
h3	4	95

RoomTable			
id	price	roomType	hotel
r1	60	single	h1
r2	90	double	h1
r3	80	single	h2
r4	120	double	h2
r5	70	single	h3
r6	90	double	h3

Tower
id
t1

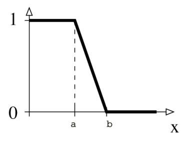
Park
id
p1
p2

DistanceTable			
id	from	to	time
d1	h1	t1	10
d2	h2	p1	15
d3	h3	p2	5

an ontology $\mathcal{O}$ [6] encompassing the following inclusion axioms

$$Park \sqsubseteq Attraction \;\; , \; Tower \sqsubseteq Attraction \;\; , \; Attraction \sqsubseteq Site$$

[6] `http://donghee.info/research/SHSS/ObjectiveConceptsOntology(OCO).html`.



$$ls(x;a,b) = \begin{cases} 1 & \text{if } x \leqslant a \\ 0 & \text{if } x \geqslant b \\ (b-x)/(b-a) & \text{if } x \in [a,b] \end{cases}$$

Fig. 1. Left shoulder function $ls(x;a,b)$.

and a set $\mathcal{A}$ of abstraction statements such as:

```
Hotel ↦ (h.id). SELECT h.id
                FROM HotelTable h

cheapPrice ↦ (h.id, r.price)[score]. SELECT h.id, r.price, cheap(r.price) AS score
                                     FROM HotelTable h, RoomTable r
                                     WHERE h.id = r.hotel
                                     ORDER BY score

closeTo ↦ (from, to)[score]. SELECT d.from, d.to closedistance(d.time) AS score
                             FROM DistanceTable d
                             ORDER BY score
```

where $cheap(p)$ is a function determining how cheap a hotel room is given its price, modelled as *e.g.* a so-called left-shoulder function (defined in Figure 1). We set $cheap(p) = ls(p;50,100)$, while $closedistance(d) = ls(d;5,25)$.

Assume now that our target concept H is $GoodHotel$, and that

- $\mathcal{E}^+ = \{GoodHotel^+(h1)[0.6], GoodHotel^+(h2)[0.8]\}$, while $\mathcal{E}^- = \{GoodHotel^-(h3)[0.4]\}$;
- $GoodHotel^+ \sqsubseteq GoodHotel$ and $GoodHotel^- \sqsubseteq GoodHotel$ occur in $\mathcal{K}$.

As illustrative example, we compute the confidence degree of

$$r : Hotel \sqcap \exists cheapPrice.\top \sqcap \exists closeTo.Attraction \sqsubseteq GoodHotel$$

i.e., a good hotel is one having a cheap price and close to an attraction. Now, it can be verified that for $\mathcal{K}' = \mathcal{K} \cup \mathcal{E}$

1. The query

$$q(h)[s] \leftarrow GoodHotel^+(h), cheapPrice(h,p)[s_1], closeTo(h,a)[s_2], Attraction(a), s = \min(s_1,s_2)$$

   has answer set $ans_{\mathcal{K}'}(q_P) = \{(h1, 0.75), (h2, 0.4)\}$ over $\mathcal{K}'$;
2. The query

$$q(h)[s] \leftarrow GoodHotel(h), cheapPrice(h,p)[s_1], closeTo(h,a)[s_2], Attraction(a), s = \min(s_1,s_2)$$

   has answer set $ans_{\mathcal{K}'}(q_D) = \{(h1, 0.75), (h2, 0.4), (h3, 0.6)\}$ over $\mathcal{K}'$;
3. Therefore, according to (9), $P = \{h1, h2\}$, while $D = \{h1, h2, h3\}$;
4. As a consequence,

$$cf(r) = \frac{0.75 \Rightarrow 0.6 + 0.4 \Rightarrow 0.8}{3} = \frac{0.6 + 1.0}{3} = 0.5333\ .$$

4 Final remarks

In this abstract we have briefly presented the core ingredients for inducing ontology inclusion axioms within the KR framework of a fuzzy DL-Lite like DL. These ingredients can be used to extend FOIL in a twofold direction: from crisp to fuzzy and from rules to inclusion axioms. Related FOIL-like algorithms reported in [10,3,9] can only learn fuzzy rules. The formal study of fuzzy ILP contributed by [5] is also relevant but less promising than our proposal in practice. Close to our application domain, [4] faces the problem of inducing equivalence axioms in a fragment of OWL corresponding to the $\mathcal{ALC}$ DL. Last, the work reported in [6] is based on an ad-hoc translation of fuzzy Łukasiewicz $\mathcal{ALC}$ DL constructs into LP and then uses a conventional ILP method to lean rules.

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